

Revisiting Bohr's principle of complementarity using a quantum device

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Bohr's principle of complementarity lies at the central place of quantum mechanics, according to which the light is chosen to behave as a wave or particles, depending on some exclusive detecting devices. Later, intermediate cases are found, but the total information of the wave-like and particle-like behaviors are limited by some inequalities. One of them is Englert-Greenberger (EG) duality relation. This relation has been demonstrated by many experiments with the classical detecting devices. Here by introducing a quantum detecting device into the experiment, we find the limit of the duality relation is exceeded due to the interference between the photon's wave and particle properties. However, our further results show that this experiment still obey a generalized EG duality relation. The introducing of the quantum device causes the new phenomenon, provides an generalization of the complementarity principle, and opens new insights into our understanding of quantum mechanics.

Bohr's principle of complementarity (BPC) has been the cornerstone of quantum theory since it was proposed in 1928 [1, 2]. This principle states that some physical objects have multiple properties, but these properties are exhibited depending on some types of exclusive detecting devices. One well-known example is the wave-particle duality, by considering a single particle in a two-way interferometer [3]. One can choose to observe the wave-like or particle-like behaviors of the particle by using different detection arrangements. Interference fringes have been observed for massive particles such as neutrons [4], electrons [5], atoms [6, 7] and molecules [8], all thought to be only particle-like before. These observations shows the unfamiliar wave-like side of these particles. In the case of light, both the anti-bunching effect and its interference fringes—associated with its particle-like and wave-like properties respective—have been previously demonstrated [9–11].

Besides these all-or-nothing situations, there actually exists some intermediate stages [12–16], where the which-path knowledge corresponding to the particle-like property is partially detected, resulting in the reduced interference visibility. This issue was first discussed by Wootters and Zurek in 1979 [12]. Later, an inequality was experimentally shown by Greenberger and Yasin in some unbalanced neutron interferometry experiments [17, 18], and theoretically derived by Jaeger *et al.* [19] and Englert [20, 21] independently. This inequality is written as

$$V^2 + D^2 \leq 1, \quad (1)$$

where V is the visibility of the interference fringes, and D is the path distinguishability of the particle, which stands for the available quantity of which-path knowledge from the system. This inequality is also known as the EG duality relation. Plenty of experiments have demonstrated

this inequality with atoms [22], nuclear magnetic resonance [23, 24], faint laser [25], and also single photons in a delayed-choice scheme [26]. Recently, this duality relation has been extended to the more general case of an asymmetric interferometer where only a single output port is considered, and this inequality still holds [27].

One of the most efficient quantum systems for testing BPC is the single photons in a Mach-Zehnder interferometer (MZI). In Ref. [26], a series of unbalanced beam splitters (BS) were randomly chosen in the MZI, including the extreme cases with reflection coefficients of $R = 0$ and 0.5 . However, we notice that beam splitters of this type are all classical devices. Mapping to the quantum BS (q-BS) scheme recently proposed by Ionicioiu and Terno [28, 29], the same results will come out when the q-BS is selected to collapse on a set of eigenstates. These eigenstates of the q-BS can be the same as the previously-mentioned classical devices.

In our experiment, the q-BS stays at the quantum superposition states of the extreme eigenstates—noted as $|a\rangle$ ($R = 0$) and $|p\rangle$ ($R = 0.5$)—corresponding to the absence and presence of a balanced BS, respectively. We introduce this q-BS into the MZI, and not only the eigenstates but also the quantum-superposition states of the q-BS are selected as the bases to collapse on at detection. The particles are single photons emitted from an InAs/GaAs self-assembled quantum dot [30, 31]. Our result shows that the EG duality relation is exceeded when some certain detecting basis of the q-BS is chosen. This exceeding is caused by the interference between the wave and particle properties of the photons. In order to derive a generalized EG duality relation, we consider both of the two orthogonal detecting bases, then we find the generalized EG duality relation holds for our results.

The experimental setup is sketched in Fig. 1(a). The single photons are split by a $50 : 50$ BS into two paths, followed by a φ phase, then combined by a q-BS. The use of the q-BS is the main difference between this setup and a regular MZI. The photons are detected by the single-photon avalanche photodiodes (APD).

As discussed in Ref. [27], we need to derive the photon

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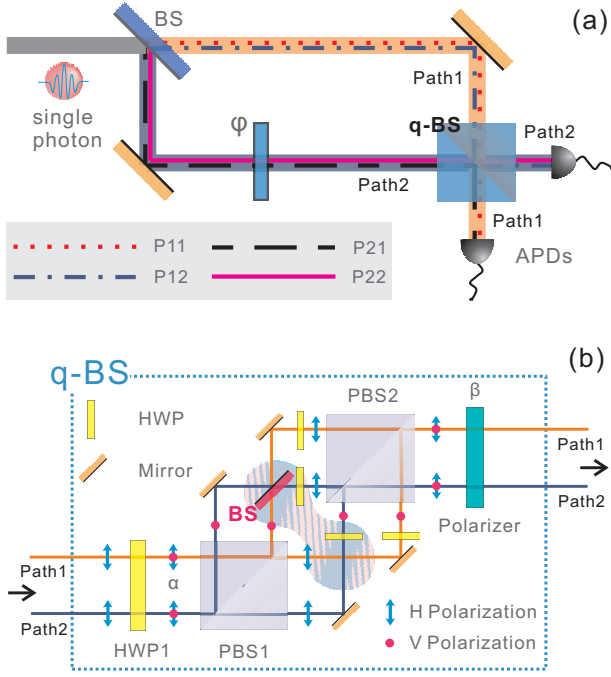


FIG. 1: (a) The MZI with a q-BS. The main difference between this setup and a regular MZI is that the second BS is replaced with a q-BS. P_{ij} ($i,j=1,2$) are the four possible sub-paths for the single photon used to define the distinguishability D . (b) The simplified setup of the q-BS. Path 1 and Path 2 are both divided into two components, which are in the quantum superposition states. Each component corresponds to an eigenstate of the photon polarization. One component constructs the closed MZI (a BS is present) and the other constructs the open MZI (no BS). PBS2 then recombines these two components, making a quantum-superposition state of the closed and open MZIs. The direction of the photon polarization before PBS1, α , controls the states of q-BS. The polarizer with a β oriented axis selects the detecting basis of the q-BS.

state after the q-BS and know the probabilities of each path taken by the photon, in order to calculate the visibility. The state of the q-BS is $|qbs\rangle = \frac{1}{\sqrt{2}}(|a\rangle + |p\rangle)$; hence we derive the photon state (before the q-BS state is detected) as

$$|\psi\rangle = \frac{1}{\sqrt{2}}|particle\rangle|a\rangle + \frac{1}{\sqrt{2}}|wave\rangle|p\rangle \quad (2)$$

according to Ionicioiu and Terno [28], with $|particle\rangle = \frac{1}{\sqrt{2}}(|1\rangle + e^{i\varphi}|2\rangle)$ corresponding to the particle state, and $|wave\rangle = e^{i\frac{\varphi}{2}}(\cos\frac{\varphi}{2}|1\rangle e^{i\delta_1} - i\sin\frac{\varphi}{2}|2\rangle e^{i\delta_2})$ corresponding to the wave state. δ_1 and δ_2 are two additional constant phases, which can be adjusted in the experiment. The q-BS state is then collapsed on an arbitrary basis $|b\rangle = \sin\beta|a\rangle + \cos\beta|p\rangle$, which means the photon state becomes $\rho = \tilde{\rho}/\text{Tr}(\tilde{\rho})$, where $\tilde{\rho} = \text{Tr}_{q-BS}(P_b|\psi\rangle\langle\psi|)$ with $P_b = |b\rangle\langle b|$ as the projection operator. Here we derive the probability that the photon takes Path 2 as $p_2(\varphi) = \text{Tr}(|2\rangle\langle 2|\rho)$. From this probability, we have the

visibility of Path 2,

$$V = \frac{p_{max} - p_{min}}{p_{max} + p_{min}}. \quad (3)$$

As shown in Fig. 1(a), each photon has four possible sub-paths to reach the APDs from the first BS (P11, P12, P21, P22). The photons that finally appear on Path 2 can come from either P12 or P22, each of which represents a totally different which-path knowledge. Assuming that the probabilities of the photons coming from P12 and P22 are respectively w_{12} and w_{22} , the distinguishability of Path 2 can be written as

$$D = |w_{12} - w_{22}|. \quad (4)$$

When the photons definitely come from either P12 or P22, then $D = 1$; when the chance that the photons are coming from either of the two paths is equal, $D = 0$. The same definitions of V and D are also found in Ref. [27], where the inequality (1) is proven to be correct for a general situation using the classical unbalanced beam splitters.

For our experiment, the q-BS is realized by using the photon polarization state as an ancilla to control the absence or presence of the BS. The simplified setup of the q-BS is shown in Fig. 1(b). Photon polarization for either path is first rotated by HWP1 (half-wave plate) in the direction of α , which corresponds to the q-BS state of $|qbs\rangle = \sin\alpha|a\rangle + \cos\alpha|p\rangle$. For this experiment, we fix this angle as $\alpha = 45^\circ$. The photons are then split by PBS1 (polarizing beam splitter) into two components. In one direction, the photons go through a closed MZI with a 50 : 50 BS; in the other direction the photons go through an open MZI with no BS. The two components are then recombined by PBS2, with the photon state at that point exactly described by Eq. (2) with $|a\rangle \leftrightarrow |V\rangle$ and $|p\rangle \leftrightarrow |H\rangle$. Note that $|H\rangle$ and $|V\rangle$ represent the horizontal and vertical polarization states of the photons, respectively. The polarizer set at the angle of β chooses the detecting basis of the q-BS.

We use the beam displacer (BD) actually to construct the MZIs instead of the regular BS or PBS, i.e., the first BS in Fig. 1(a) and the BS, PBS1 and PBS2 in Fig. 1(b). The BD moves the extraordinary beam to a parallel path separated from the ordinary beam by 4 mm, making the MZI more stable than a traditional BS system. Simplified sketches of the setup are given in Fig. 1, as they illustrate the setup better than diagrams depicting all the elements of the setup and aid in understanding. A more detailed description of the setup can be found in our previous work [32].

To measure the visibility, we leave both paths in Fig. 1(a) unblocked, count the photon numbers detected by the APDs, then calculate the probability that the photon takes Path 2, i.e., $p_2(\varphi)$. The results are shown in Fig. 2. The solid lines are the theoretical fits corresponding to each set of experimental data. Fig. 2(a) is the $\beta = 0$ case, where the q-BS state is detected on

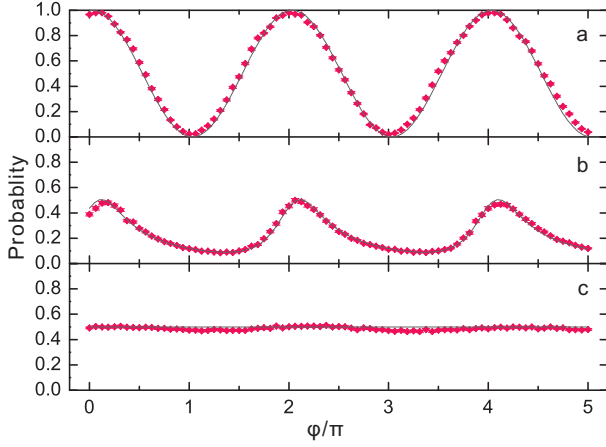


FIG. 2: Probability that the photon takes Path 2. (a), (b) and (c) correspond to the cases of $\beta = 0$, $\frac{3\pi}{16}$ and $\frac{\pi}{2}$, respectively. The solid lines are the corresponding theoretical fits for each case.

the basis of $|b\rangle = |p\rangle$, which is the eigenstate associated with the closed MZI. Therefore, the photons behave as a wave, and the visibility (shown in Fig. 3(a)) of the interference fringe reaches 0.961 ± 0.004 . This result coincides with the classical-BS-experiment result found in Ref. [26]. Fig. 2(c) corresponds to the $\beta = \frac{\pi}{2}$ case. Similarly, the q-BS state is detected on the other eigenstate $|b\rangle = |a\rangle$, which is associated with the open MZI. Thus, the photons behave as particles. The result is also the same as in the classical BS case. However, $\beta = \frac{3\pi}{16}$ for Fig. 2(b), so the detecting basis here is a quantum-superposition state, which is related to the MZI staying in both a closed and an opened state. The visibility in this case is 0.707 ± 0.017 . The photons behave as a quantum superposition of wave and particle, which is well illustrated by the expression describing the photons' state ρ , i.e., $C_1(\sin\beta|particle\rangle + \cos\beta|wave\rangle)$ (where C_1 is a coefficient). This phenomenon does not have a counterpart in the classical BS experiment. The differences between the experimental and theoretical values are caused by the counting statistics, the imperfection of the optical glasses, the dark and background counts, and the tiny instability of the MZIs.

To measure the distinguishability D , we first block Path 1 after the BS in Fig. 1(a) and detect the number of photons coming from P22 (N_{22}), then block Path 2, and detect the photon number from P12 (N_{12}). Hence, the distinguishability of Path 2 can be calculated using $D = \frac{|N_{12} - N_{22}|}{N_{12} + N_{22}}$ according to Eq. (4). The result is shown in Fig. 3(b) with larger dots, and the smaller-dot line is the theoretical simulation. When $\beta = 0$ (the closed MZI), then $D = 0.045 \pm 0.024$ and no which-path knowledge is available. However, when $\beta = \frac{\pi}{2}$ (the open MZI), then $D = 0.97751 \pm 0.0038$ and full which-path knowledge is detected. This result is in accord with the wave-like and particle-like behavior of the photons previously discussed. For these all-or-nothing cases, the q-BS collapses

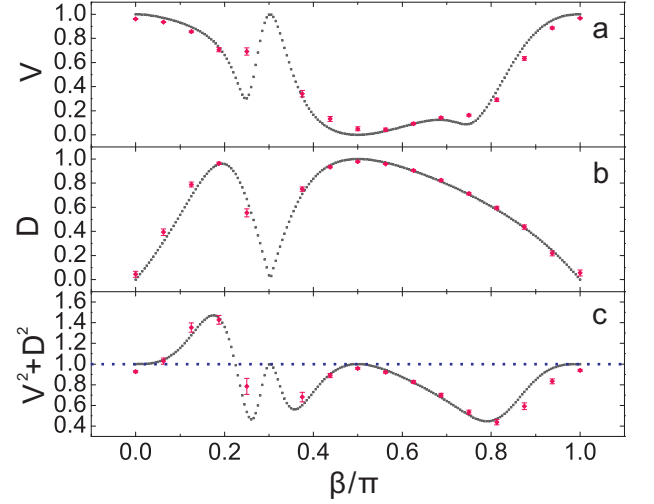


FIG. 3: (a) The visibility V , (b) The path distinguishability D , and (c) $V^2 + D^2$. The dashed line in (c) is the limit of the EG duality relation (1), which is exceeded in this situation.

on the eigenstates, which means these situations give the same results as the classical BS experiment; the inequality (1) holds, and the upper bound is reached (See in Fig. 3(c)). On the other hand, in the quantum intermediate case of $\beta = \frac{3\pi}{16}$, the value of $V^2 + D^2$ goes beyond the limit of the EG duality relation (1, the blue dashed line in Fig. 3(c)) by 10 deviations to reach 1.428 ± 0.043 . This result coincides with the results from the theoretical simulation.

This exceeding of the EG duality relation is caused by the quantum superposition of the photons' wave and particle states—or the interference between them—introduced by the q-BS and a quantum intermediate detecting basis. To illustrate this point and derive a generalized EG duality relation, we combine the corresponding photon counts of the two orthogonal bases related to β and $\beta + \frac{\pi}{2}$, then calculate $V_g^2 + D_g^2$ in the same way. The forms of V_g and D_g are the same as V and D , respectively. However, the photon counts and the meanings are different. The former ones correspond to the sum of the counts of two orthogonal bases, and describe the behavior of photons in these two cases as a whole; the wave-particle interference becomes an internal effect here. On the other hand, the later ones describe the behavior of photons in a single basis case. We find that the generalized inequality ($V_g^2 + D_g^2 \leq 1$) holds for our results, shown in Fig. 4(a). The solid line is the theoretical simulation. To further analyze this combination process, we calculate the final state of the photon after the combination, found to be $C_2(\sin^2\alpha|particle\rangle\langle particle| + \cos^2\alpha|wave\rangle\langle wave|)$, where C_2 is a coefficient. This state is a classical mixture of the wave and particle properties, and is independent of the chosen orthogonal basis pair (defined by β). However, the state is related to the parameter α , which determines the state of the q-BS and also the probabilities of the photon going through the closed or open MZIs. $V_g^2 + D_g^2$

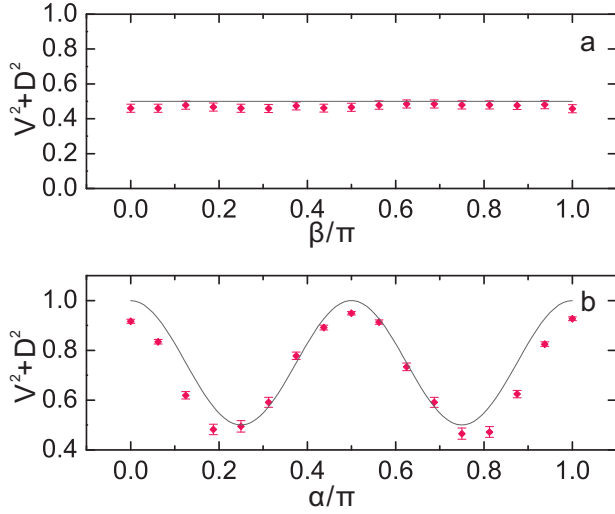


FIG. 4: $V_g^2 + D_g^2$ after combination of the photon numbers of two orthogonal-basis cases with (a) varying β and fixed $\alpha = \frac{\pi}{4}$ and (b) varying α and arbitrary β . The generalized EG duality relation holds for these results.

is calculated to be $\sin^4\alpha + \cos^4\alpha$, which is not larger than 1; when $\alpha = \frac{\pi}{4}$, then $V_g^2 + D_g^2 = 0.5$. We have also measured $V_g^2 + D_g^2$ using various values of α , with the result shown in Fig. 4(b), which further proves our previous discussions. There is a systematic error in Fig. 4, which may caused by the dark and background counts, the decoherence processes, the imperfection of optical glasses and the imprecision of experimental parameters.

Actually, the violation of BPC—and specifically the EG duality relation—has been declared by Afshar *et al.* [33], who believe that quantum mechanics is not correct, but others disagree with this interpretation [34–37], and the debate continues. We must note here that our experiment is completely unrelated to the Afshar experiment. Even though our results exceed the EG duality relation, our experiment as a whole is in accord with quantum

theory and is only a small extension of BPC, i.e., the classical detecting devices are replaced with the quantum devices for our experiment. In the original BPC, the detecting devices can only be in the classical states, which are each related to the properties that can be shown. In contrast, the detecting devices can exist in the quantum-superposition states in our extension by using the quantum control [28]. This small change makes the originally exclusive properties of the object appears to be quantum-superposed, allowing for the limit of the EG relation duality to be exceeded.

Besides experiments in wave-particle duality, there are many other well known experiments whose results form the foundation of quantum mechanics: the Bell-inequality experiments [38–40], the Kochen-Specker-inequality experiments [41–43], and so on. The new concept of using a quantum device could also be introduced into these experiments, potentially allowing new phenomena to appear, which could further our understanding of quantum mechanics.

In conclusion, we introduce a q-BS, proposed in Ref. [28], into the unbalanced MZI used in Ref. [26], selecting some quantum-superposition states of the q-BS as the collapsing bases to detect the q-BS's states. Following the definitions of visibility and distinguishability used in Ref. [27], we find the limit of the EG duality relation is exceeded. We conclude that this result is caused by the interference between the wave and particle properties of the photons. After we combine the corresponding photon numbers of two mutually orthogonal collapsing bases of q-BS, the wave-particle interference becomes an internal effect, then the generalized EG duality relation holds. This work is entirely within standard quantum theory, but opens up a new way for people to understand the quantum world by replacing the classical devices with quantum ones.

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- [1] Bohr, N. Das Quantenpostulat und die neuere Entwicklung der Atomistik. *Naturwissenschaften* **16**, 245 (1928).
 - [2] Bohr, N. The quantum postulate and the recent development of atomic theory. *Nature* **121**, 580 (1928).
 - [3] Feynman, R. P., Leighton, R. B. & Sands, M. L. *Lectures on Physics* (Addison Wesley, Reading, MA, 1963).
 - [4] Summhammer, J., Badurek, G., Rauch, H., Kischko, U. & Zeilinger, A. Direct observation of fermion spin superposition by neutron interferometry. *Phys. Rev. A* **27**, 2523 (1983).
 - [5] Tonomura, A., Endo, J., Matsuda, T., Kawasaki, T. & Ezawa, H. Demonstration of single-electron buildup of an interference pattern. *Am. J. Phys.* **57**, 117 (1989).
 - [6] Carnal, O. & Mlynek, J. Young's Double-Slit Experiment with Atoms: A Simple Atom Interferometer. *Phys. Rev. Lett.* **66**, 2689 (1991).
 - [7] Keith, D. W., Ekstrom, C. R., Turchette, Q. A. & Pritchard, D. E. An Interferometer for Atoms. *Phys. Rev. Lett.* **66**, 2693 (1991).
 - [8] Arndt, M. *et al.* Wave-particle duality of C60 molecules. *Nature* **401**, 680 (1999).
 - [9] Grangier, P., Roger, G. & Aspect A. Experimental Evidence for a Photon Anticorrelation Effect on a Beam Splitter: A New Light on Single-Photon Interferences. *Europhys. Lett.* **1**, 173 (1986).
 - [10] Braig, C., Zarda, P., Kurtsiefer, C. & Weinfurter, H. Experimental demonstration of complementarity with single photons. *Appl. Phys. B* **76**, 113 (2003).
 - [11] Jacques, V. *et al.* Single-photon wavefront-splitting interference. *Eur. Phys. J. D* **35**, 561 (2005).
 - [12] Wootters, W. K. & Zurek, W. H. Complementarity in the double-slit experiment: Quantum nonseparability and a quantitative statement of Bohr's principle. *Phys. Rev. D* **19**, 473 (1979).

- [13] Rauch, H. & Summhammer, J. Static versus time-dependent absorption in neutron interferometry. *Phys. Lett. A* **104**, 44 (1984).
- [14] Summhammer, J., Rauch, H. & Tuppinger, D. Stochastic and deterministic absorption in neutron-interference experiments. *Phys. Rev. A* **36**, 4447 (1987).
- [15] Buks, E., Schuster, R., Heiblum, M., Mahalu, D. & Umansky, V. Dephasing in electron interference by a ‘which-path’ detector. *Nature* **391**, 871 (1998).
- [16] Dürr, S., Nonn, T. & Rempe, G. Origin of quantum-mechanical complementarity probed by a ‘which-way’ experiment in an atom interferometer. *Nature* **395**, 33 (1998).
- [17] Greenberger, D. M. & Yasin, A. Simultaneous wave and particle knowledge in a neutron interferometer. *Phys. Lett. A* **128**, 391 (1988).
- [18] Mandel, L. Coherence and indistinguishability. *Opt. Lett.* **16**, 1882 (1991).
- [19] Jaeger, G., Shimony, A. & Vaidman, L. Two interferometric complementarities. *Phys. Rev. A* **51**, 54 (1995).
- [20] Englert, B. G. Fringe Visibility and Which-Way Information: An Inequality. *Phys. Rev. Lett.* **77**, 2154 (1996).
- [21] Englert, B. G. & Bergou, J. A. Quantitative quantum erasure. *Opt. Commun.* **179**, 337 (2000).
- [22] Dürr, S., Nonn, T. & Rempe, G. Fringe Visibility and Which-Way Information in an Atom Interferometer. *Phys. Rev. Lett.* **81**, 5705 (1998).
- [23] Peng, X. *et al.* An interferometric complementarity experiment in a bulk nuclear magnetic resonance ensemble. *J. Phys. A: Math. Gen.* **36**, 2555 (2003).
- [24] Peng, X. *et al.* Quantification of complementarity in multiqubit systems. *Phys. Rev. A* **72**, 052109 (2005).
- [25] Schwindt, P. D. D., Kwiat, P. G. & Englert, B. G. Quantitative wave-particle duality and nonerasing quantum erasure. *Phys. Rev. A* **60**, 4285 (1999).
- [26] Jacques, V. *et al.* Delayed-Choice Test of Quantum Complementarity with Interfering Single Photons. *Phys. Rev. Lett.* **100**, 220402 (2008).
- [27] Li, L., Liu, N. L. & Yu, S. X. Duality relations in a two-path interferometer with an asymmetric beam splitter. arXiv:1202.3326v1.
- [28] Ionicioiu, R. & Terno, D. R. Proposal for a quantum delayed-choice experiment. *Phys. Rev. Lett.* **107**, 230406 (2011).
- [29] Schirber, M. Focus: Another Step Back for Wave-Particle Duality. *Physics* **4**, 102 (2011).
- [30] Tang, J. S. *et al.* Direct observation of single InAs/GaAs quantum dot spectrum without mesa or mask. *Phys. E* **41**, 797 (2009).
- [31] Li, C. F., Tang, J. S., Li, Y. L. & Guo, G. C. Experimentally witnessing the initial correlation between an open quantum system and its environment. *Phys. Rev. A* **83**, 064102 (2011).
- [32] Tang, J. S. *et al.* Realization of quantum Wheeler’s delayed-choice experiment (submitted).
- [33] Afshar, S. S., Flores, E., McDonald, K. F. & Knoes, E. Paradox in Wave-Particle Duality. *Foundations of Physics* **37**, 295 (2007).
- [34] Steuernagel, O. Afshar’s Experiment Does Not Show a Violation of Complementarity. *Foundations of Physics* **37**, 1370 (2007).
- [35] Jacques, V. *et al.* Illustration of quantum complementarity using single photons interfering on a grating. *New Journal of Physics* **10**, 123009 (2008).
- [36] Georgiev, D. D. Single Photon Experiments and Quantum Complementarity. *Progress in Physics* **2**, 97 (2007).
- [37] Georgiev, D. D. Quantum histories and quantum complementarity. *ISRN Mathematical Physics* **2012**, 327278 (2012).
- [38] Bell, J. S. On the Einstein Podolsky Rosen paradox. *Physics* **1**, 195 (1964).
- [39] Clauser, J. F., Horne, M. A., Shimony, A. & Holt, R. A. Proposed experiment to test local hidden-variable theories. *Phys. Rev. Lett.* **23**, 880 (1969).
- [40] Freedman, S. J. & Clauser, J. F. Experimental test of local hidden-variable theories. *Phys. Rev. Lett.* **28**, 938 (1972).
- [41] Kochen, S. & Specker, E. P. The problem of hidden variables in quantum mechanics. *J. Math. Mech.* **17**, 59 (1967).
- [42] Mermin, N. D. What’s wrong with these elements of reality? *Phys. Today* **43**, 9 (1990).
- [43] Huang, Y. F., Li, C. F., Zhang, Y. S., Pan, J. W. & Guo, G. C. Experimental test of the Kochen-Specker theorem with single photons. *Phys. Rev. Lett.* **90**, 250401 (2003).